x,y,H are the observation points on the strip,
x_0,y_0,H are the source points on the strip.

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Equivalent Series Inductivity of a Narrow Transverse Slit in Microstrip

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Abstract—**The series inductivity introduced by a narrow transverse slit in a microstrip transmission line has been evaluated theoretically, and a simple formula for this inductivity is presented. Experimental results for slits of different depth obtained with the resonant ring method compare well with theory. Applications of such a slit include the fine tuning of the electrical length of stubs and the compensation of excess capacitances at discontinuities.**

I. Introduction

The insertion of a narrow transverse slit or notch into a microstrip transmission line leads to a local concentration of the magnetic field which can be described in terms of an equivalent series inductivity. Fig. 1(a) shows a microstrip of width w containing a slit of depth a and width b, centered about the $z = 0$ plane. Some current lines and lines of magnetic field have been drawn to demonstrate the effect of the slit on the propagating quasi-TEM fields.

The equivalent series inductivity ΔL (Fig. 1(b)) is independent of the slit width b as long as the slit is narrow, i.e., b is smaller than the substrate thickness h and much smaller than the transmission line wavelength λ_g. In the following, the normalized series inductivity $\Delta L/h$ will be calculated as a function of w/h of the microstrip and the relative depth a/w of the slit. It is assumed that the substrate is nonmagnetic ($\mu_r = 1$). Dispersion and capacitive effects will be neglected.

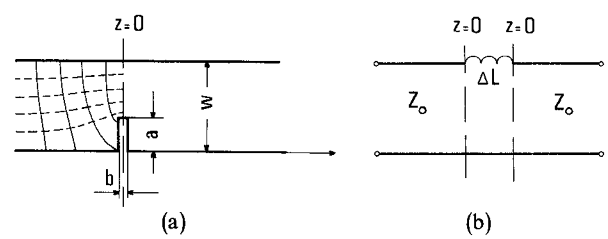

Fig. 1. (a) Narrow transverse slit in a microstrip line. Some current lines (dotted) and magnetic field lines (solid) demonstrate the effect of the slit. (b) Equivalent series inductivity in the $z = 0$ plane.

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II. Analysis of the Slit

The parameters of many transmission-line discontinuities can be calculated with reasonable accuracy by assuming that they create a perturbation in the form of a dipole field when excited by the incident field. Wheeler [1] has outlined some basic principles of this method. His equivalent volume concept will be applied in the following study.

The main difficulty in the calculation of the slit inductivity resides in the fact that, on the one hand, the application of the equivalent volume concept calls for a uniform excitation of the slit, but, on the other hand, the slit is situated partly in the highly nonuniform fringing field of the microstrip.

To overcome this problem, the microstrip line is replaced

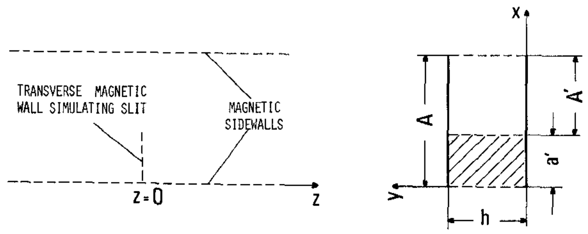

Fig. 2. Transverse magnetic wall simulating the narrow transverse slit in a parallel-plate microstrip model.

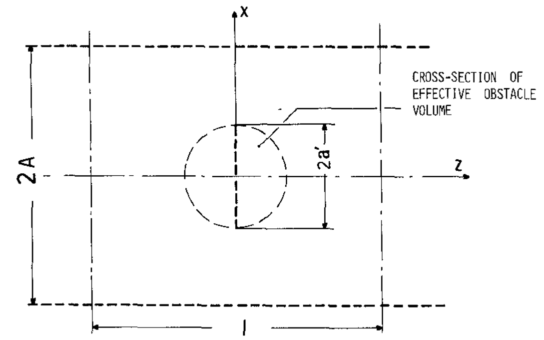

Fig. 3. Symmetrical magnetic-wall model of microstrip containing a narrow transverse slit.

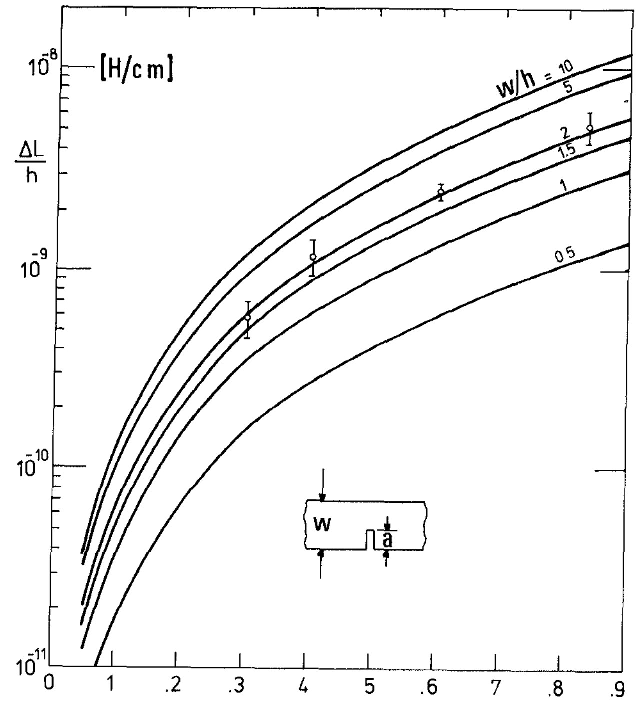

Fig. 4. Normalized series inductivity of narrow transverse slit in microstrip, calculated for several values of w/h, as a function of the relative slit depth a/w. The inductivity is independent of the substrate permittivity. Some experimental values for $w/h = 2$ are shown for comparison.

by a parallel-plate waveguide model with magnetic sidewalls. The dimensions of this model have been given by Leighton and Milnes [2]. It describes the quasi-TEM properties of the microstrip very well, contains uniform fields, and presents well-defined simple boundary conditions. It has been used successfully by Wolff *et al.* [3], Hoefer and James [4], and Hoefer and Chattopadhyay [5] to evaluate microstrip discontinuity parameters.

The microstrip line of characteristic impedance Z_0 and effective permittivity ε_{eff} is thus represented by a model of height h and equivalent width A. In the $z = 0$ plane, however, where the current lines are more concentrated, the fields are confined to a narrower equivalent cross section of width A':

$$A' = \frac{h}{Z_0'} \sqrt{\frac{\mu_0}{\varepsilon'_{\text{eff}} \varepsilon_0}}. \tag{1}$$

In this expression, Z' and $\varepsilon'_{\text{eff}}$ are the parameters of a uniform microstrip line of width $w' = w - a$, and of height h.

If the slit is narrow, its width b may be neglected, and the effect of the slit can be modeled by inserting a transverse magnetic wall of zero thickness into the microstrip equivalent, as shown in Fig. 2. The penetration depth a' of the transverse wall is the difference between the widths of the equivalent cross sections:

$$a' = A - A'. \tag{2}$$

For convenient analysis, this structure is transformed into a symmetrical configuration by adding its image about the $x = 0$ plane, as shown in Fig. 3. We thus obtain a model which is related by duality to the capacitive structure containing a conductive obstacle, as discussed by Wheeler [1] to illustrate the concept of effective volume.

Accordingly, the inductivity of the structure in Fig. 3 is increased by the insertion of the magnetic obstacle in its center. If the length l of the line section is much greater than the width $2a'$ of the obstacle, but still small compared with the guided wavelength, the relative increase in inductivity can be expressed as a volume ratio:

$$\frac{\Delta L_s}{L_s} = \frac{V_{\text{ob}}}{V_l} \tag{3}$$

where V_{ob} is the effective volume of the obstacle, and V_l the volume of the model section. The effective volume V_{ob} is the volume of a cylinder of height h and diameter $2a'$. If the inductivity L_s itself is expressed in terms of the geometry of the microstrip model, the absolute increase ΔL_s becomes, after normalization with respect to the substrate thickness h,

$$\frac{\Delta L_s}{h} = \frac{\mu_0 \pi}{4} \left(\frac{a'}{A}\right)^2. \tag{4}$$

Finally, the series inductivity ΔL in the original microstrip is twice the inductivity introduced in the symmetrical model. We obtain

$$\frac{\Delta L}{h} = 2 \frac{\Delta L_s}{h} = \frac{\mu_0 \pi}{2} \left(\frac{a'}{A}\right)^2 \tag{5}$$

where a'/A can be interpreted as the relative penetration depth of the transverse magnetic wall into the model shown in Fig. 2. Equation (5) can be expressed directly in terms of characteristic impedances and effective dielectric constants by writing

$$\frac{a'}{A} = 1 - \frac{Z_0}{Z_0'} \sqrt{\frac{\varepsilon_{\text{eff}}}{\varepsilon'_{\text{eff}}}} = 1 - \frac{Z_{0_{\text{air}}}}{Z'_{0_{\text{air}}}} \tag{6}$$

where the primed quantities characterize a uniform microstrip line of width $w = w - a$. It turns out that the slit inductivity is independent of the substrate permittivity, since it can be expressed in terms of the characteristic impedances of air-filled microstrip lines of widths w and $w' = w - a$, respectively; their height h being the same as that of the corresponding dielectric-filled microstrips. The characteristic impedances can easily be obtained from the slit geometry by using available design diagrams or formulas for microstrip.

Equation (5) has been evaluated and presented graphically in Fig. 4 as a function of the relative slit depth a/w for several values of the parameter w/h. Some measured values for $w/h = 2$ are included for comparison.

III. Limitations of the Analysis

It is both necessary and interesting to consider the limitations of the expression for the slit inductivity. One limiting case is that of a very small slit width b, where the displacement current across the slit can no longer be neglected. One way to evaluate the importance of the displacement current is to estimate the series capacitance of the slit and compare it with the slit inductance.

Benedek and Silvester [6] have calculated the equivalent capacitances of microstrip gaps. From their results, the series capacitance of a slit can be obtained in the following way: If C_{12} is the series capacitance of a gap in a microstrip of width $2a$ and substrate thickness h, and if the gap spacing is b, then the series capacitance of a slit of depth a and width b is approximately $C_s = C_{12}/2$.

The following example will demonstrate the quantities involved in a practical situation. Consider a slit of relative depth $a/w = 0.5$ in a 50-Ω line on a 25-mil-thick alumina substrate ($w/h = 1$; $\varepsilon_r = 9.6$). At 10 GHz, the inductance of the slit is $j3.7$ Ω. If the slit is only 2.5 mils wide, its series capacitance is approximately $-j320$ Ω, about a hundred times larger than the slit inductance and thus negligible in most cases.

The other limiting case is that of a very wide slit. The slit then becomes a set of two nonsymmetrical impedance steps separated by a short section of transmission line of high characteristic impedance. The slit width for which this transmission line effect occurs depends on the dielectric constant. As a rule of thumb, the slit inductivity is independent of b as long as b is smaller than the substrate thickness h.

Since the discontinuity was calculated under the assumption that it created a dipole field, the validity of the analysis is further restricted by the condition that the slit depth a should be small compared with the strip width w. However, measurements show that the simple formula obtained above gives sufficiently accurate results even for $a/w = 0.83$. It is therefore thought that the formula is satisfactory for values of a/w ranging from 0 to 0.9, which should cover most practical situations.

IV. Experimental Results

Measurements of ΔL were made in a resonant ring on RT-Duroid substrate with $\varepsilon_r = 2.35$ and $h = 1.5$ mm. The ring was circular, with a mean diameter of 20 cm and an aspect ratio $w/h = 2$. A 1-mm-wide slit was cut into the ring with relative depths of 0.3, 0.4, 0.6, and 0.833 successively, and the change in resonant frequencies of the ring was measured according to the method described in [5].

In Fig. 4, these measurements are compared with theoretical values obtained with (5). The circles show average values taken over the first six resonances of the ring, while the bars indicate the standard deviation of measured values from this average.

It can be seen that the relatively simple formula (5) agrees very well with measurements, even for very deep slits and low substrate permittivity causing considerable fringing.

V. Conclusion

The equivalent series inductivity of narrow transverse slits in microstrip has been calculated using the concept of volume ratios. The microstrip was replaced by a parallel-plate model with magnetic sidewalls, and the effect of the slit was simulated by a transverse magnetic wall of zero thickness, penetrating into the model. The formula obtained for the slit inductivity is independent of the dielectric constant of the substrate material and of the width of the slit as long as the latter dimension is roughly between one and one tenth of the substrate thickness. Experimental values obtained with the resonant ring method confirm the validity of the theoretical expression for the slit inductivity.

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